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AN ANALYSIS OF FRAME SYNCHRONIZATION VIA UP/DOWN COUNTERS.(U)

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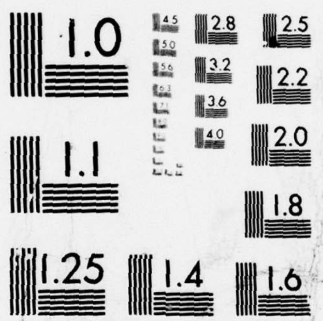

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TECHNICAL NOTE NO. 17-78

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To determine that a digital communications system is properly synchronized for transmitting data, ~~an~~ an up/down counter is used. A mathematical analysis of the counter is described; the analysis is in computer implementable form.

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AN ANALYSIS OF FRAME SYNCHRONIZATION VIA UP/DOWN COUNTERS

August 1978

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FOREWORD

The Defense Communications Engineering Center (DCEC) Technical Notes (TN's) are published to inform interested members of the defense community regarding technical activities of the Center, completed and in progress. They are intended to stimulate thinking and encourage information exchange; but they do not represent an approved position of policy of DCEC, and should not be used as authoritative guidance for related planning and/or further action.

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I. INTRODUCTION

In transmitting blocks of data through a communications system, ensuring that the system is in synchronization is vital. Each frame or block of data contains a header of information. These headers are checked to ensure that the system is still in sync. There are two possible ways that the header may not be correct: first, some bits may be transmitted wrong or secondly, the system could be out of sync. Once sync has been lost, some corrective action has to be taken,during which the system may be down. In order to ensure that the system is not taken down for bursts of transmission errors an up/down counter is used to monitor its behavior.

The counter has N+1 slots labelled 0,1,...,N and a pointer that is moved up or down according to whether the header is correct or not. Usually, if the header is correct the pointer is moved up one slot - unless it is at N in which case it remains there. The number of slots that the pointer is moved down is the subject of this investigation. We analyze the case where the pointer is moved down $K(\leq N)$ slots when the header is wrong. We assume that if the pointer is within K slots of zero, then the pointer is moved to zero when an incorrect header appears. Once at zero, the pointer remains there and the system is assumed to be out of sync. The focus of the study is the random variable which describes the elapsed time from starting at slot N until the pointer is at zero for the first time. This random variable represents the time until checking the system to see if it is still in sync once the counter has been reset.

If p is the probability an arriving header is wrong, and q=1-p that it is right; then the system can be modelled as a Markov chain and the

analysis can be carried out in a straightforward manner. In section II, this is done; section III contains some numerical examples. Finally, section IV gives some concluding statements.

This problem was generated because of conversations the author has had with Dr. Stan Lorens of R710 and Mr. Walter Roehr of R740. Dr. Lorens pointed out that the Naval Research Lab (NRL) has written a specification for an Advanced Narrowband Digital Voice Terminal (ANDVT) [1]. The linear predictive coder in this terminal will be capable of extracting frame sync. The frame synchronization shall consist of two modes: sync acquisition and sync maintenance. The sync maintenance function is the one that is analyzed in this technical note. For the ANDVT it consists of a counter numbered 0 to 15. When a correct sync bit is received, the counter is incremented by one. If the check bit is incorrect it is decremented by two. Once the counter reaches zero, sync has to be reacquired. So the ANDVT sync maintenance is a special case (N=15, K=2) of our general counter.

Dr. Lorens stated that to the best of his knowledge no analysis had been done to determine if the NRL specification was satisfactory. Mr. Roehr added that it would be desirable to have the capability to work at more general parameters than just N=15 and K=2 as in the NRL specification. These conversations have resulted in the mathematical analysis and computer implementation of these results presented in this technical note. The author is grateful to Dr. Lorens and Mr. Roehr for bringing this problem to his attention.

As a closing comment to this section we point a related piece of work by Smith [2]. He considered the more general problem of acquiring and maintaining sync. In this paper a more extensive reference list of related work on this problem is given.

II. MATHEMATICAL ANALYSIS

As we mentioned in the preceding section, the system can be modelled as a Markov chain. For a discussion of these types of stochastic processes, see [3], [4], [5] or [6]. We will use some definitions from Markov chains. The reader should refer to the texts cited above for an explanation of unfamiliar terms.

Let X_n be the random variable representing the position of the pointer just after the header was observed for the nth time. We adopt the convention that the observation of the header and the subsequent changing of the placement of the pointer occur simultaneously. The random variables X_n, $n=1,2,\ldots,$ can take on the values $0,1,\ldots,N$. Since the probability that the header is correct, q, does not depend on n, the value X_n takes on only depends on X_{n-1} and as such forms a Markov chain. In Markov chains the most important element which governs how the chains progress in time is called the probability transition matrix, usually denoted by P. The i,jth entry of this matrix gives the probability that $X_n=j$ given that $X_{n-1}=i$ for all i and j. For the Markov chain being considered here the probability transition matrix, P, is given by

$$P = \begin{array}{cc} & \begin{array}{ccccccccc} 0 & 1 & 2 & \cdots & K & K+1 & \cdots & N-1 & N \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ \vdots \\ K \\ K+1 \\ \vdots \\ N-1 \\ N \end{array} & \left[\begin{array}{ccccccccc} 1 & 0 & 0 & \cdots & & & & & 0 \\ p & 0 & q & 0 & & \cdots & & & 0 \\ p & 0 & 0 & q & & & & & \\ & & & & & & & & \\ p & 0 & & & & q & & & 0 \\ 0 & p & 0 & & & 0 & q & & 0 \\ & & & & & & & & \\ 0 & & & p\;0 & & & & & 0\;q \\ 0 & & & & p\;0 & & & 0 & 0\;q \end{array} \right] \end{array} . \tag{1}$$

The zero state is called an absorbing state in that once this Markov chain goes there, it never leaves. If $q=1$, the state N would also be absorbing, but in the application to up/down counters, q is probably close to but not equal to 1.

Now we are in position to define the random variable that we wish to study. Suppose at some instant, denoted by 0, we observe the system and the pointer is pointing at the ith slot, i.e., $X_0=i$. We want to study the behavior of the system as it goes to zero for the first time. We define Y_i to be the random variable representing the time (in number of observations of the header) until the pointer is at zero for the first time, given that the initial observation of the pointer is at the ith slot. We are really interested in Y_N but need the more general definition for the development. Note that Y_N represents the time until checking for sync once the counter has been reset. Let us define

$$
\begin{aligned}
f_{i,n} &= Pr\{Y_i=n\} \\
&= Pr\{X_n=0, X_{n-1}\neq 0, X_{n-2}\neq 0, \ldots, X_1\neq 0 \mid X_0=i\}.
\end{aligned}
\tag{2}
$$

We are interested in studying $f_{N,n}$, the probability of the pointer being at zero for the first time at the nth observation of the header given that it was initially set at the Nth slot. For a general K,p, and N the exact solution for $f_{N,n}$ is difficult; some special cases will be discussed in section IV, but a simple recursion relation allows one to iteratively solve for $f_{N,n}$. For n=1 we have ($f_{0,0}=1$, $f_{i,0}=0$ $i\geq 1$)

$$
f_{i,1} = \begin{cases} p & i=1,2,\ldots K \\ 0 & i=K+1,K+2,\ldots,N \end{cases}
\tag{3}
$$

and for $n \geq 2$ (note $f_{0,n} \equiv 0$ here) we have

$$f_{i,n} = \begin{cases} qf_{i+1,n-1} + pf_{0,n-1} & i=1,2,\ldots,K \\ qf_{i+1,n-1} + pf_{i-K,n-1} & i=K+1,K+2,\ldots,N-1 \\ qf_{N,n-1} + pf_{N-K,n-1} & i=N. \end{cases} \tag{4}$$

The development of equation (4) is as follows: if the initial observation of the pointer is at the ith slot and we want to go to zero at the nth observation for the first time, we consider what happens at the first observation. For $i=1,2,\ldots,K$ we can go to the $(i+1)$st slot with probability q (header correct); once at the $(i+1)$st step we have to go to zero for the first time in $n-1$ observations. The latter quantity is $f_{i+1,n-1}$ by the homogeneity of the Markov chain. With probability p we would go to zero from these slots but $n \geq 2$ in equation (4), and since we stay at zero there is no contribution to $f_{i,n}$ for this event.

For $i=K+1,K+2,\ldots,N-1$ we can move up one slot with probability q to $i+1$ or drop back K to $i-K$ with probability p, and then go to zero in $(n-1)$ observations. Finally, for $i=N$ we stay at slot N with probability q or drop back to $N-K$ with probability p. The form of equation (4) makes it ideally suitable for implementation on the computer.

The exact solution $f_{N,n}$ for a general K is difficult, but a form of solution can be obtained via generating functions. Define for $i=1,2,\ldots,N$ and $|z| \leq 1$ $(\Pi_0(z)=1)$

$$\Pi_i(z) = \sum_{n=0}^{\infty} f_{i,n} z^n; \tag{5}$$

then from the first part of equation (4) we have

$$\Pi_i(z)=qz\Pi_{i+1}(z)+pz; \tag{6}$$

the last term on the right hand side of equation (6) comes from $f_{i,1}$. For $i=K+1,\ldots,N-1$

$$\Pi_i(z)=qz\Pi_{i+1}(z)+pz\Pi_{i-K}(z) \tag{7}$$

and finally for $i=N$ in equation (4), one gets

$$(1-qz)\Pi_N(z)=pz\Pi_{N-K}(z). \tag{8}$$

The solution technique for $\Pi_i(z)$ is a standard one for these types of equations and follows the development given in [5]. Equation (7) suggests a trial solution of the form $\Pi_i(z)=\lambda^i(z)$. Substituting this form of solution in equation (7) one gets

$$qz\lambda^{K+1}(z)-\lambda^K(z)+pz=0. \tag{9}$$

Let $\lambda_r(z)$, $r=1,2,\ldots,K+1$, be the roots for this equation, then the general solution to equation (7) is given by

$$\Pi_i(z)=\sum_{r=1}^{K+1}\alpha_r(z)\lambda_r^i(z), \tag{10}$$

where $\alpha_r(z)$, $r=1,2,\ldots,K+1$, are to be determined.

These K+1 unknowns may be found from the K equations given in equation (6) and the one given by equation (8). Using these equations and equation (10) one gets the following K+1 equations in $\alpha_r(z)$; for $i=1,2,\ldots,K$

$$\sum_{r=1}^{K+1}\alpha_r(z)[\lambda_r^i(z)(1-qz\lambda_r(z))]=pz \tag{11}$$

and

$$\sum_{r=1}^{K+1} \alpha_r(z)\{(1-qz)\lambda_r^N(z)-pz\lambda_r^{N-K}(z)\} = 0. \tag{12}$$

Thus, equations (9), (11) and (12), can be used to solve the problem. For practical application, this solution becomes cumbersome although the results for K=1 provides some insight and is now given. For K=1, there are two roots of equation (9); they are

$$\lambda_1(z) = \frac{1+\sqrt{1-4pqz^2}}{2qz} \tag{13}$$

and

$$\lambda_2(z) = \frac{1-\sqrt{1-4pqz^2}}{2qz}. \tag{14}$$

Since

$$\Pi_i(z)=\alpha_1(z)\lambda_1^i(z)+\alpha_2(z)\lambda_2^i(z),$$

using equation (11) for K=1, one finds

$$1 = \alpha_1(z)+\alpha_2(z). \tag{15}$$

Using this result in equation (12), $\alpha_2(z)$ is given by

$$\alpha_2(z) = \frac{(1-qz)\lambda_1^N(z)-pz\lambda_1^{N-1}(z)}{pz(\lambda_2^{N-1}(z)-\lambda_1^{N-1}(z))+(1-qz)(\lambda_1^N(z)-\lambda_2^N(z))}. \tag{16}$$

These results may be used to find an analytic expression for the expected value of Y_N, $E(Y_N)$. That is,

$$E(Y_N)=\Pi_N'(1)= \frac{q(p^N-q^N)+Np^N(q-p)}{p^N(2p-1)(q-p)}. \tag{17}$$

For a general K, a simple expression for even $E(Y_N)$ is difficult to obtain.

Although one may use equations (6), (7), and (8) to recursively find all the moments of Y_N, we find the mean, $E(Y_N)$, and variance, $Var(Y_N)$, of Y_N from $\Pi_N(z)$. Using the basic definitions we have

$$E(Y_N) = \Pi_N'(1)$$

and

$$Var(Y_N) = \Pi_N''(1)+\Pi_N'(1)-\Pi_N'(1)^2.$$

Let $\Pi_i \equiv \Pi_i'(1)$, then using equations (6) and (7) we have

$$E_i = 1+qE_{i+1} \quad : \quad i=1,2,\ldots,K \tag{18}$$

and

$$E_i = 1+qE_{i+1}+pE_{i-K} \quad : \quad i=K+1,K+2,\ldots,N-1. \tag{19}$$

Note we have used the fact that $\Pi_i(1)=1$. From equations (18) and (19) one sees that E_i for $i=2,3,\ldots,N$ can be expressed in terms of E_1 so let us define

$$E_i = C(i,1)E_1+C(i,2); \tag{20}$$

that is, E_i is a constant, $C(i,1)$, times E_1 plus another constant $C(i,2)$, with $C(1,1)$ and $C(1,2)=0$. Using equation (18) we have

$$qC(i,1) = \begin{cases} C(i-1,1) & i=2,3,\ldots,K+1 \\ C(i-1,1)-pC(i-1-K,1) & i=K+2,\ldots,N \end{cases} \tag{21}$$

and

$$qC(i,2) = \begin{cases} C(i-1,2)-1 & i=2,3,\ldots,K+1 \\ C(i-1,2)-1-pC(i-1-K,2) & i=K+2,\ldots,N \end{cases} \tag{22}$$

From equation (8) we have

$$pE_N = 1+pE_{N-K}, \tag{23}$$

which can be used with equations (19) and (20) to find

$$E_1 = \frac{1+p(C(N-K,2)-C(N,2))}{p(C(N,1)-C(N-K,1))}. \tag{24}$$

Thus, one can recursively evaluate $C(i,1)$ and $C(i,2)$, using equation (24) to find E_1 and then set

$$E_N \equiv E(Y_N) = C(N,1)E_1+C(N,2).$$

In order to find $Var(Y_N)$ one has to find $\Pi_N''(1)$; this can be done in the same manner we found $\Pi_N'(1)$. For $i=1,2,\ldots,N$, let

$$F_i \equiv \Pi_i''(1) = D(i,1)F_1+D(i,2),$$

obviously, $D(i,1)=1$ and $D(i,2)=0$. Proceeding as before we have

$$qD(i,1) = \begin{cases} D(i-1,1) & :i=2,3,\ldots,K+1 \\ D(i-1,1)-pD(i-1-K,1) & :i=K+2,\ldots,N \end{cases} \tag{25}$$

and

$$qD(i,2) = \begin{cases} D(i-1,2)-2qE_i & :i=2,3,\ldots,K+1 \\ D(i-1,2)-pD(i-1-K,2)-Z_i & :i=K+2,\ldots,N \end{cases} \tag{26}$$

where $Z_i=2qE_i+2pE_{i-1-K}$. Again, using equation (8) we have

$$F_1 = \frac{p[D(N-K,2)-D(N,2)] + 2qE_N+2pE_{N-K}}{p[D(N,1)-D(N-K,1)]} \tag{27}$$

and $F_N = D(N,1)F_1+D(N,2)$.

We have given a recursive method for easily computing $E(Y_N)$, $Var(Y_N)$ and the complete probability density function of Y_N. In the next section we give some numerical examples using these results.

III. SOME NUMERICAL EXAMPLES

In this section we give three numerical examples using the results presented in section II. The first is given in Table I; the probability density function of the number of observations of the header until the pointer goes from N=15 to zero for the first time is given. For that table, K=2 (i.e., the NRL specification) and p was set equal to .6, .7, .8, and .9. The reason for the large values of p is to ensure that non-zero values of $f_{15,n}$ are basically given when $n \leq 40$. Since K=2 we have $f_{15,n}=0$ for $n \leq 7$; that is, it takes at least 8 observations for the system to drop to zero. It is interesting to point out the oscillatory nature of $f_{15,n}$ for a fixed value of p. Consider p=.8, then $f_{15,9}=.26844$, $f_{15,10}=.05369$ and $f_{15,11}=.16106$; thus these probabilities depend on K and how the sample function of Y_{15} may proceed to zero.

Tables II and III give values of the mean and variance of the time until checking for out sync for N=15 and fixed different values of p and K. One of the most interesting facts that can be seen in these figures is that

$$E(Y_{15})^2 \approx Var(Y_{15}), \qquad (28)$$

(where $\approx$ means approximately)for p small. This fact was not true for the parameters used in Table I. The fact that the parameters selected for Tables II and III are more representative of real world values tends to make the result given by equation (28) more promising for future work. This idea will be discussed in the next section.

Another interesting and important fact that can be seen is that the performance of this up/down counter is extremely sensitive to the values of p and K. From Table II, for K=2 when p changes from .01 to .5, the expected time to check to see if the system is out of sync changes from 1.09×10^{15} to 275. If a header is arriving every .0225 seconds, then this number corresponds to checking for sync either every 18.7×10^{6} years or every 6.1875 seconds, Similarly, the p fixed and K varying we have the same sort of results; see p=.05 in Table II. These results indicate that setting the specification at a given value of K is extremely critical and should be studied in more depth.

TABLE I. PROBABILITY DENSITY FUNCTION FOR TIME UNTIL CHECKING IF OUT OF SYNC (N=15, K=2)

$f_{N,n}$	p			
n	.6	.7	.8	.9
8	.01680	.05765	.16777	.43047
9	.05375	.13836	.26844	.34437
10	.02150	.04151	.05369	.03444
11	.05375	.11414	.16106	.11192
12	.09374	.15627	.15247	.05458
13	.03956	.04950	.03221	.00577
14	.06525	.08748	.06073	.01156
15	.09392	.10130	.04967	.00496
16	.04176	.03384	.01107	.00055
17	.05748	.04909	.01689	.00089
18	.07520	.05252	.01296	.00036
19	.03495	.01835	.00302	.00004
20	.04348	.02378	.00407	.00006
21	.05373	.02432	.00302	.00002
22	.02592	.00883	.00073	.00000
23	.03023	.01003	.00091	
24	.03596	.01056	.00066	
25	.01790	.00396	.00017	
26	.01998	.00453	.00019	
27	.02312	.00441	.00014	
28	.01183	.00170	.00004	
29	.01278	.00187	.00004	
30	.01449	.00180	.00003	
31	.00759	.00071	.00001	
32	.00800	.00076	.00001	
33	.00892	.00072	.00001	
34	.00477	.00029	.00000	
35	.00493	.00030		
36	.00343	.00028		
37	.00296	.00012		
38	.00301	.00012		
39	.00328	.00011		
40	.00182	.00005	.00000	.00000
$E(Y_{15})$	18.209	13.603	10.880	9.109
$Var(Y_{15})$	52.539	18.828	7.080	2.219

TABLE II. EXPECTED TIME UNTIL CHECKING IF OUT OF SYNC FOR N=15*

K \ P	.01	.05	0.1	0.2	0.3	0.4	0.5
1	8.87×10^{29}	1.78×10^{19}	2.9×10^{14}	2.39×10^{9}	1.45×10^{6}	6.48×10^{3}	2.40×10^{2}
2	1.09×10^{15}	1.70×10^{9}	3.27×10^{6}	6.40×10^{3}	2.36×10^{2}	5.47×10^{1}	2.75×10^{1}
3	6.62×10^{9}	7.68×10^{5}	1.17×10^{4}	2.28×10^{2}	4.69×10^{1}	2.25×10^{1}	1.46×10^{1}
4	2.07×10^{7}	1.97×10^{4}	9.08×10^{2}	6.81×10^{1}	2.46×10^{1}	1.43×10^{1}	1.01×10^{1}
5	6.94×10^{5}	2.64×10^{3}	2.46×10^{2}	3.7×10^{1}	1.68×10^{1}	1.07×10^{1}	7.8×10^{0}
6	9.67×10^{4}	7.62×10^{2}	1.14×10^{2}	2.51×10^{1}	1.28×10^{1}	8.54×10^{0}	6.4×10^{0}
7	3.74×10^{4}	3.89×10^{2}	7.35×10^{1}	2.01×10^{1}	1.11×10^{1}	7.8×10^{0}	6.07×10^{0}
8	4.68×10^{3}	1.71×10^{2}	4.60×10^{1}	1.51×10^{1}	8.6×10^{0}	5.95×10^{0}	4.51×10^{0}
9	2.55×10^{3}	1.16×10^{2}	3.57×10^{1}	1.28×10^{1}	7.55×10^{0}	5.33×10^{0}	4.13×10^{0}
10	1.78×10^{3}	9.22×10^{1}	3.04×10^{1}	1.16×10^{1}	7.08×10^{0}	5.11×10^{0}	4.03×10^{0}
11	1.39×10^{3}	7.84×10^{1}	2.73×10^{1}	1.09×10^{1}	6.86×10^{0}	5.04×10^{0}	4.01×10^{0}
12	1.14×10^{3}	6.95×10^{1}	2.53×10^{1}	1.06×10^{1}	6.76×10^{0}	5.02×10^{0}	4.00×10^{0}
13	9.79×10^{2}	6.34×10^{1}	2.39×10^{1}	1.04×10^{1}	6.71×10^{0}	5.01×10^{0}	4.00×10^{0}
14	8.61×10^{2}	5.9×10^{1}	2.30×10^{1}	1.02×10^{1}	6.69×10^{0}	5.00×10^{0}	4.00×10^{0}
15	1.00×10^{2}	2.0×10^{1}	1.0×10^{1}	5.00×10^{1}	3.33×10^{0}	2.50×10^{0}	2.00×10^{0}

*This time is the number of sync maintenance checks that have to be made. To convert this to time one has to multiply this number by the expected time to transmit a frame.

TABLE III. VARIANCE OF TIME UNTIL CHECKING IF OUT OF SYNC FOR N=15

K \ P	.01	.05	0.1	0.2	0.3	0.4	0.5
1	7.86×10^{59}	3.17×10^{38}	8.5×10^{28}	5.69×10^{18}	2.09×10^{12}	4.14×10^{7}	3.83×10^{4}
2	1.21×10^{28}	2.87×10^{18}	1.4×10^{13}	4.07×10^{7}	4.57×10^{4}	1.34×10^{3}	1.91×10^{2}
3	4.39×10^{14}	5.9×10^{11}	1.4×10^{8}	4.57×10^{5}	1.2×10^{3}	1.69×10^{2}	4.79×10^{1}
4	4.30×10^{14}	3.87×10^{8}	8.1×10^{5}	3.39×10^{3}	2.81×10^{2}	6.56×10^{1}	2.38×10^{1}
5	4.8×10^{11}	6.91×10^{6}	5.65×10^{4}	8.94×10^{2}	1.24×10^{2}	3.66×10^{1}	1.51×10^{1}
6	$9,35 \times 10^{9}$	5.71×10^{5}	1.14×10^{4}	3.92×10^{2}	7.3×10^{1}	2.42×10^{1}	1.02×10^{1}
7	1.39×10^{9}	1.46×10^{5}	4.52×10^{3}	2.27×10^{2}	4.77×10^{1}	1.65×10^{1}	7.2×10^{0}
8	2.19×10^{7}	2.8×10^{4}	1.78×10^{3}	1.44×10^{2}	3.79×10^{1}	1.53×10^{1}	7.4×10^{0}
9	6.51×10^{6}	1.28×10^{4}	1.05×10^{3}	1.04×10^{2}	2.91×10^{1}	1.16×10^{1}	5.4×10^{0}
10	3.16×10^{6}	7.85×10^{3}	7.39×10^{2}	8.17×10^{1}	2.34×10^{1}	9.4×10^{0}	4.5×10^{0}
11	1.91×10^{6}	5.54×10^{3}	5.71×10^{2}	6.79×10^{1}	2.01×10^{1}	8.4×10^{0}	4.15×10^{0}
12	1.29×10^{6}	4.25×10^{3}	4.69×10^{2}	5.89×10^{1}	1.81×10^{1}	7.9×10^{0}	4.05×10^{0}
13	9.47×10^{5}	3.45×10^{3}	4.01×10^{2}	5.29×10^{1}	1.70×10^{1}	7.7×10^{0}	4.01×10^{0}
14	7.31×10^{5}	2.91×10^{3}	3.53×10^{2}	4.89×10^{1}	1.63×10^{1}	7.6×10^{0}	4.00×10^{0}
15	9.9×10^{4}	3.80×10^{3}	9.0×10^{2}	2.0×10^{1}	7.77×10^{1}	3.8×10^{0}	2.0×10^{0}

IV. CONCLUSIONS

We have given a mathematical analysis for the operation of an up/down counter that is used to monitor the sync behavior of data in a telecommunications system. We close this technical note with a few remarks concerning the work and some possible extensions.

First, from a historical point of view the formulation of the problem presented here is closely related to the classical gambler's ruin problem. This problem is thoroughly discussed in Feller [3]. In that problem K=1 and the state N is also an absorbing state. We have not searched the literature on the gambler's ruin problem thoroughly enough to ensure that more work on our problem has not been done. If further work in this area is to be done one should look at the literature more closely.

The results that we gave were in the form of recursive relationships. As such they are easily implemented on the computer. Two problems arise with this type of solution: first, since no analytic expression (except for K=1) was given for $f_{N,n}$, $E(Y_N)$, or $Var(Y_N)$, the exact dependency on N, K, and p can only be ascertained by drawing conclusions from the numerical examples. It would be desirable to have an analytic expression so that more concrete conclusions can be drawn; although this appears to be a formidable task.

The second problem with the recursive solution is one of numerical accuracy. The smallest value of p that we have considered in the numerical examples is p= 0.01, for $K \leq 5$ and N=15; this gave values of $E(Y_{15})$ and $Var(Y_{15})$ on the order of 10^7 and 10^{14} or greater. Numbers of this magnitude are subject

to the numerical accuracies of the computer one is dealing with. If closed form analytic expressions were given, this problem might be avoided as was done for the K=1 case.

For N=K, a little thought will convince the reader that

$$f_{N,n} = q^{n-1}p \qquad :n=1,2,\ldots, \tag{29}$$

from which

$$E(Y_N) = p^{-1} \tag{30}$$

and

$$Var(Y_N) = qp^{-2}. \tag{31}$$

These results along with equation (17) allow us to give some bounds on $E(Y_N)$ as a function of K. For N and p fixed $E(Y_N)$ is minimum when K=N and maximum when K=1. Thus, equation (17) is an upper bound and equation (30) a lower bound on the expected number of observations until the pointer is at zero for the first time.

We remarked in section III that for the values of p used in Tables II and III, the mean time until out of sync was approximately equal to the square root of the variance. Since some numerical problems could be encountered in evaluating the performance of the counter for p's smaller than .001, some sort of approximation for these cases should be considered. One possible approach is along the lines of a diffusion approximation as presented by Feller [2] for the gambler's ruin problem. It would be interesting to see if that development could be extended to our problem.

Finally, it should be reemphasized that the table given in section III clearly shows that the setting of K for a fixed value of p is extremely critical. Just changing the value of K one slot can make several orders of magnitude difference in the results one gets. Extreme care should be taken when fixing K. For instance, for the ANVDT we had N=15, K=2. If the probability of the check bit being received wrong (p) changes from .01 to .5 this corresponds to having to reacquire sync every 18.7×10^6 years to every 6.1875 seconds.

It is hoped that the results presented in this paper may be used to provide some insight into the performance of up/down counters used to ensure the system is maintaining sync.

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